

Universes inside a Λ black hole

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We address the question of universes inside a Λ black hole which is described by a spherically symmetric globally regular solution to the Einstein equations with a variable cosmological term $\Lambda_{\mu\nu}$, asymptotically $\Lambda g_{\mu\nu}$ as $r \rightarrow 0$ with Λ of the scale of symmetry restoration. Global structure of spacetime contains an infinite sequence of black and white holes, vacuum regular cores and asymptotically flat universes. Regular core of a Λ white hole models the initial stages of the Universe evolution. In this model it starts from a nonsingular nonsimultaneous big bang, which is followed by a Kasner-type anisotropic expansion. Creation of a mass occurs mostly at the anisotropic stage of quick decay of the initial vacuum energy. We estimate also the probability of quantum birth of baby universes inside a Λ black hole due to quantum instability of the de Sitter vacuum.

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Introduction- The idea of a de Sitter core replacing a black hole singularity goes back to the 60-s papers by Sakharov who suggested $p = -\varepsilon$ as an equation of state at superhigh densities [1], and by Gliner who interpreted $p = -\varepsilon$ as a vacuum equation of state and suggested that it could be a final state in a gravitational collapse [2].

In the 80-s several solutions have been obtained by direct matching de Sitter metric inside to Schwarzschild metric outside of a spacelike junction surface Σ_0 of the Planckian thickness $\delta \sim l_{Pl} \sim 10^{-33}\text{cm}$ [3,4]. Global structure of spacetime in this case is shown in Fig.1.

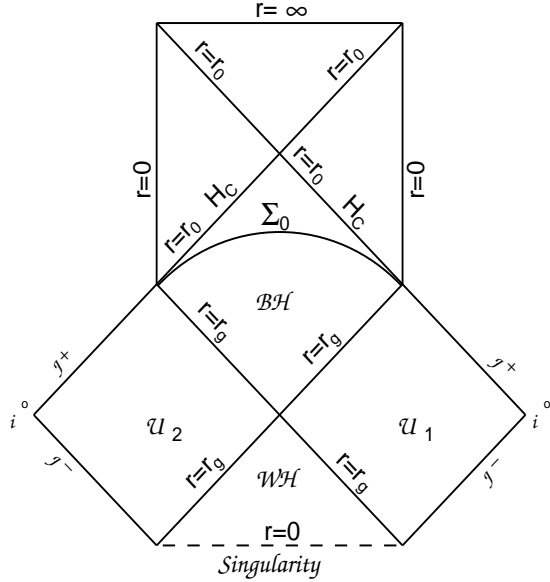


FIG. 1. Penrose-Carter diagram for the case of the direct de Sitter-Schwarzschild matching.

The idea of a baby Universe inside a black hole has been proposed by Farhi and Guth (FG) in 1987 [4] as the idea of creation of a universe in the laboratory starting from a false vacuum bubble in the Minkowski space. FG studied an expanding spherical de Sitter bubble separated by a thin wall from the outside region of the Schwarzschild geometry. The global structure of spacetime in this case (Fig.1) implies that the expanding bubble must be associated with an initial spacelike singularity which clearly represents a singular initial value configuration. Therefore Farhi and Guth concluded that the initial singularity would be an unavoidable obstacle to creation of a universe in the laboratory [4].

In 1988 Poisson and Israel have analyzed de Sitter-Schwarzschild transition and found that the spacetime geometry can be self-regulatory and describable semi-classically down to a few Planckian radii by the Einstein equations with a source term representing vacuum polarization effects [5]. They found also that the Cauchy horizon must exist in this geometry (H_C at the Fig.1).

In 1989 arising a new universe inside a black hole has been considered by Frolov, Markov, and Mukhanov (FMM) [6] in the context of the hypothesis that the curvature is limited by the Planckian scale and at this scale the equation of state becomes $p = -\varepsilon$. The difference of FMM from FG approach is that FG assumption of existence of a global Cauchy surface may be violated, due to the existence of the Cauchy horizon [5], which implies the absence of a global Cauchy surface.

In 1990 Farhi, Guth and Guven studied the model in which the initial bubble is small enough to be produced without initial singularity [7]. A small bubble classically could not become a universe - instead it would reach a

maximum radius and then contract. FGG investigated the possibility that quantum effects allow the bubble to tunnel into the larger bubble of the same mass which for an external observer disappears beyond the black hole horizon, whereas on the inside the bubble would classically evolve to become a new universe [7].

Both FG and FMM models are based on matching the Schwarzschild and de Sitter metrics using thin shell approach which implies that the whole dynamical evolution from the equation of state $\varepsilon = p = 0$ to $p = -\varepsilon_{Pl}$ occurs within a junction layer of the Planckian thickness $\delta \sim l_{Pl} \sim 10^{-33}\text{cm}$. As a result the matched metrics typically have a jump at the junction surface.

The general case of a smooth de Sitter-Schwarzschild transition, i.e., of a distributed density profile, has been qualitatively addressed in the paper by Frolov, Markov, and Mukhanov in 1990 [8]. They mentioned two possibilities: or arising a new macroscopic closed universe either creation of a white hole in a new asymptotically flat universe which lies in the absolute future with respect to the original asymptotically flat universe.

The exact analytic solution describing de Sitter-Schwarzschild transition in general case of a distributed density profile, has been found by one of us [9] in a simple semiclassical model for density profile due to vacuum polarization in a spherically symmetric gravitational field [10]. This solution belongs to the class of solutions to the Einstein equations with the source term such that $T_r^r = T_t^t$; $T_\theta^\theta = T_\phi^\phi$. The stress-energy tensor with such an algebraic structure describes a spherically symmetric vacuum invariant under boosts in the radial direction [9] and represents the extension of the Einstein cosmological term $\Lambda g_{\mu\nu}$ to the spherically symmetric r -dependent cosmological tensor Λ_μ^μ [11].

In the case of de Sitter-Schwarzschild transition it connects in a smooth way two vacuum states: de Sitter vacuum $T_{\mu\nu} = (8\pi G)^{-1}\Lambda g_{\mu\nu}$ replacing a singularity at the origin and Minkowski vacuum $T_{\mu\nu} = 0$ at infinity. This corresponds to r -dependent cosmological term evolving from $\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}$ as $r \rightarrow 0$ (with Λ of the scale of symmetry restoration in the origin [12]) to $\Lambda_{\mu\nu} = 0$ as $r \rightarrow \infty$.

In the Schwarzschild coordinates the metric is given by

$$ds^2 = \left(1 - \frac{R_g(r)}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{R_g(r)}{r}} - r^2 d\Omega^2 \quad (1)$$

where $d\Omega^2$ is the line element on the unit two-sphere. The function $R_g(r)$ represents an r -dependent gravitational radius

$$R_g(r) = 8\pi G \int_0^r \rho(x) x^2 dx = 2GM(r) \quad (2)$$

A density profile $\rho(r) = (8\pi G)^{-1}\Lambda_t^t(r)$ should be a smooth function providing the proper asymptotic behaviour of $R_g(r)$: quick vanishing as $r \rightarrow \infty$ to guarantee finiteness of a mass

$$M = 4\pi \int_0^\infty \rho(x) x^2 dx < \infty; \quad R_g(r \rightarrow \infty) = r_g, \quad (3)$$

where $r_g = 2GM$ and M is the Schwarzschild mass, and proper asymptotics as $r \rightarrow 0$ [5,8,9]

$$R_g(r \rightarrow 0) = \frac{r_0^3}{r_0^2} \quad (4)$$

where r_0 is the de Sitter horizon defined by

$$r_0^2 = \frac{3}{\Lambda} = \frac{3c^2}{8\pi G\rho_0} \quad (5)$$

Here ρ_0 is the vacuum density and $\Lambda = \Lambda_t^t(0)$ is the value of the cosmological constant at the origin.

For any density profile satisfying conditions (3)-(4), the metric (1) describes a globally regular de Sitter-Schwarzschild geometry, asymptotically Schwarzschild as $r \rightarrow \infty$, and asymptotically de Sitter as $r \rightarrow 0$ [10,13]. For cosmological term $\Lambda_{\mu\nu}$ with the algebraic structure $\Lambda_t^t = \Lambda_r^r$; $\Lambda_\theta^\theta = \Lambda_\phi^\phi$, responsible for this geometry, the inflationary equation of state is satisfied by the radial pressure $p_r^\Lambda = -\Lambda_r^r$, while the tangential pressure $p_\perp^\Lambda = -\Lambda_\theta^\theta = -\Lambda_\phi^\phi$ is calculated from the conservation equation $\Lambda_{;\nu}^{\mu\nu} = 0$, giving the equation of state for cosmological tensor Λ_μ^ν [11]

$$p_r^\Lambda = -\rho^\Lambda; \quad p_\perp^\Lambda = p_r^\Lambda + \frac{r}{2} \frac{dp_r^\Lambda}{dr} \quad (6)$$

The cosmological term $\Lambda_{\mu\nu}$ corresponds to a spherically symmetric vacuum $T_{\mu\nu}^\Lambda(r) = (8\pi G)^{-1}\Lambda_{\mu\nu}(r)$ with the variable density and pressure. It belongs to the Type I in the classification by Hawking and Ellis [14]. For any density profile decreasing monotonically ($d\rho/dr \leq 0$ everywhere) it satisfies the weak energy condition $T_{\mu\nu}u^\mu u^\nu \geq 0$ for any timelike vector u^ν , which holds if $\rho \geq 0$, $\rho + p_k \geq 0$ ($k = 1, 2, 3$) [14]. With the above restriction on a density profile $T_{\mu\nu}^\Lambda$ satisfies also the dominant energy condition $T^{00} \geq |T^{ab}|$ for each a, b , which holds if $\rho \geq 0$, $-\rho \leq p_k \leq \rho$. These conditions imply that the local energy density is non-negative and each component of the pressure never exceeds the energy density. The strong energy condition $(T_{\mu\nu} - g_{\mu\nu}T/2)u^\mu u^\nu \geq 0$, which for Type I holds if $\rho + p_k \geq 0$, $\rho + \sum p_k \geq 0$, is violated (i.e. a gravitational acceleration changes sign) at the surface of zero gravity defined by $2\rho + r d\rho/dr = 0$.

In the range of masses $M \geq M_{crit} \simeq 0.3M_{Pl}\sqrt{\Lambda_{Pl}/\Lambda}$ the de Sitter-Schwarzschild spacetime has two horizons, an event horizon $r = r_+$ and an internal Cauchy horizon $r = r_-$, and the metric (1) describes a Λ black hole (Λ BH) [10,11]. Its global structure is presented in Fig.2. It contains an infinite sequence of black and white holes, whose singularities are replaced with future and past regular cores \mathcal{RC} , and asymptotically flat universes \mathcal{U} [10]. The Penrose-Carter diagram Fig.2 is plotted in coordinates related to the photon radial geodesics. The surfaces \mathcal{J}^- and \mathcal{J}^+ represent their past and future infinities. The event horizons $r = r_+$ and the Cauchy horizons

$r = r_-$ are formed by the outgoing and ingoing radial photon geodesics $r_{\pm} = \text{const.}$

It is evident from the Penrose-Carter diagram Fig.2 that inside a Λ BH there exists an infinite number of vacuum-dominated asymptotically flat universes in the future of Λ white holes. Geodesic structure of de Sitter-Schwarzschild space-time shows that the possibility of travelling into other universes through a black hole interior, discussed in the literature for the case of Reissner-Nordström and Kerr geometry ([15] and references therein), exists also in the case of a Λ BH [16].

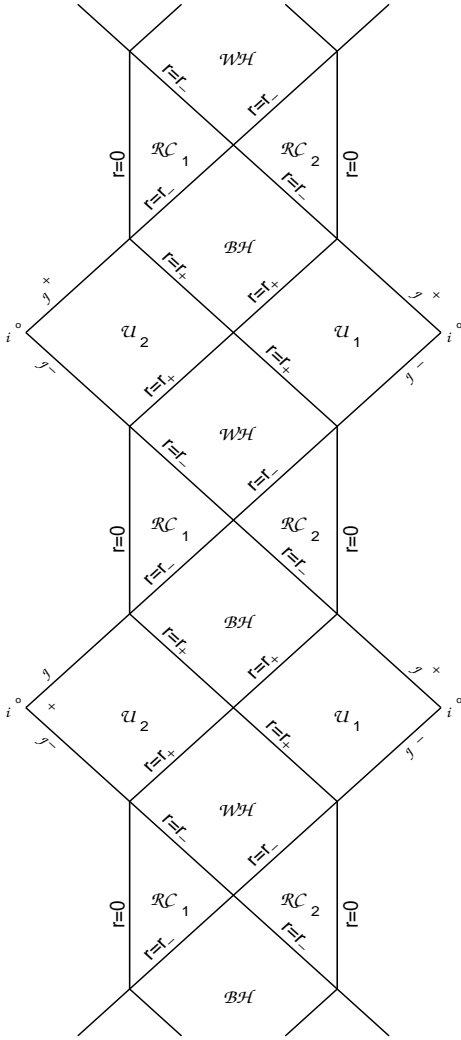


FIG. 2. Penrose-Carter diagram for a Λ black hole.

It is widely known that the interiors of white holes can be described locally as cosmological models (see, e.g., [17]). In the case of a Schwarzschild white hole it starts from the spacelike singularity $r = 0$ (see Fig.3).

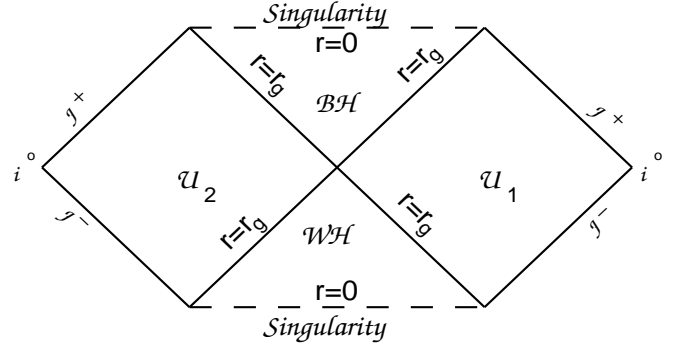


FIG. 3. Penrose-Carter diagram for the Schwarzschild geometry.

Replacing a Schwarzschild singularity with the regular core \mathcal{RC} transforms the spacelike singular surfaces $r = 0$, both in the future of \mathcal{BH} and in the past of \mathcal{WH} , into the timelike regular surfaces $r = 0$ (see Fig.2). In a sense, this rehabilitates a white hole, whose existence in a singular version has been forbidden by the cosmic censorship [18].

A cosmological model related to a Λ white hole corresponds to an asymptotically flat vacuum-dominated cosmology with the de Sitter origin governed by the time-dependent cosmological term $\Lambda_{\mu\nu}$ (segment $\mathcal{RC}, \mathcal{WH}, \mathcal{U}$ in the Fig.2). The Λ WH (more precisely its \mathcal{RC} region) models thus the initial stages of a nonsingular cosmology with the inflationary origin.

In this paper we address two questions: First is the vacuum-dominated cosmological model with the variable cosmological term $\Lambda_{\mu\nu}$, related to Λ WH. Second is question of baby universes inside a Λ BH. The case of direct de Sitter-Schwarzschild matching (Fig.1) clearly corresponds to arising of a closed or semiclosed world inside a black hole [6,8]. We estimate the probability of this event in the case of global structure Fig.2 as a result of quantum instability of the de Sitter vacuum near the surface $r = 0$. We consider also the case when some admixture of strings or quintessence is present in the initial fluctuation near $r = 0$ in the vacuum-dominated system governed by $\Lambda_{\mu\nu}$. We show that in such a case a multiple quantum birth of open or flat baby universes is possible, which are causally disjoint from each other.

Λ WH model for a nonsimultaneous big bang-

To investigate a Λ WH together with its past regular core \mathcal{RC} and future asymptotically flat universe \mathcal{U} , we introduce the Finkelstein coordinates, related to radial geodesics of nonrelativistic test particles at rest at infinity. They are given by

$$c\tau = \pm ct \pm \int \sqrt{\frac{R_g(r)}{r} + f(R)} \frac{dr}{1 - \frac{R_g(r)}{r}} \quad (7)$$

$$R = ct + \int \sqrt{\frac{r}{R_g(r)}} \frac{\sqrt{1+f(R)}dr}{1 - \frac{R_g(r)}{r}} \quad (8)$$

Here $f(R)$ is an arbitrary function satisfying the condition $1+f(R) > 0$. The lower sign in (7) is for outgoing geodesics corresponding to the case of an expansion.

The metric (1) transforms into the Lemaitre metric

$$ds^2 = c^2 d\tau^2 - e^{\lambda(R,\tau)} dR^2 - r^2(R,\tau) d\Omega^2 \quad (9)$$

with

$$e^{\lambda(R,\tau)} = \frac{R_g(r(R,\tau))}{r(R,\tau)} \quad (10)$$

Coordinates R, τ are the Lagrange (comoving) coordinates of a test particle, and r is its Euler radial coordinate (luminosity distance). In the case of outgoing geodesics the (R, τ) coordinates with the lower sign in (7), map the segment $\mathcal{RC}, \mathcal{WH}, \mathcal{U}$, i.e. ΛWH with its regular core \mathcal{RC} and its external universe \mathcal{U} .

For the metric (9) the Einstein equations reduce to [17]

$$8\pi G p_r = \frac{1}{r^2} (e^{-\lambda} r'^2 - 2r\ddot{r} - \dot{r}^2 - 1) \quad (11)$$

$$8\pi G p_\perp = \frac{e^{-\lambda}}{r} \left(r'' - \frac{r'\lambda'}{2} \right) - \frac{\dot{r}\dot{\lambda}}{2r} - \frac{\ddot{\lambda}}{2} - \frac{\dot{\lambda}^2}{4} - \frac{\ddot{r}}{r} \quad (12)$$

$$8\pi G \rho = -\frac{e^{-\lambda}}{r^2} (2rr'' + r'^2 - rr'\lambda') + \frac{1}{r^2} (r\dot{r}\dot{\lambda} + \dot{r}^2 + 1) \quad (13)$$

$$8\pi G T_t^r = \frac{e^{-\lambda}}{r} (2\dot{r}' - r'\dot{\lambda}) = 0 \quad (14)$$

Here the dot denotes differentiation with respect to τ and the prime with respect to R . The component T_t^r of the stress-energy tensor vanishes in the comoving reference frame, and Eq.(14) is integrated giving [19]

$$e^\lambda = \frac{r'^2}{1+f(R)} \quad (15)$$

Putting (15) into (11), we obtain the equation of motion in the form

$$\dot{r}^2 + 2r\ddot{r} + \kappa p_r r^2 = f(R) \quad (16)$$

This cosmological model belongs to the Lemaitre class of spherically symmetric models with anisotropic fluid [20]. The dynamics of our model is governed by cosmological tensor $\Lambda_{\mu\nu}$ which in this case is time-dependent.

Near the surface $r = 0$ the metric (9) transforms into the FRW form for any $f(R)$. It reads

$$ds^2 = c^2 d\tau^2 - a^2(\tau)(d\chi^2 + \sin^2 \chi d\Omega^2) \quad (17)$$

with the de Sitter scale factor $a(\tau) \sim \cosh(H_0\tau)$ for $f(R) < 0$, $a(\tau) \sim \sinh(H_0\tau)$ for $f(R) > 0$, $a(\tau) \sim \exp(H_0\tau)$ for $f(R) = 0$, where H_0 is the Hubble parameter corresponding to the initial value of Λ .

In this paper we present numerical results for the case of $f(R) = 0$ ($\Omega = 1$). For numerical integration of the equation of motion we adopt the density profile in the form [9]

$$\rho(r) = \rho_0 \exp\left(-\frac{r^3}{r_0^2 r_g}\right) \quad (18)$$

The characteristic scale of de Sitter-Schwarzschild space-time is $r_* = (r_0^2 r_g)^{1/3}$, and we normalize r to this scale introducing the dimensionless variable ξ by $r = r_* \xi$. The behaviour of pressures in this case is shown in Fig.4.

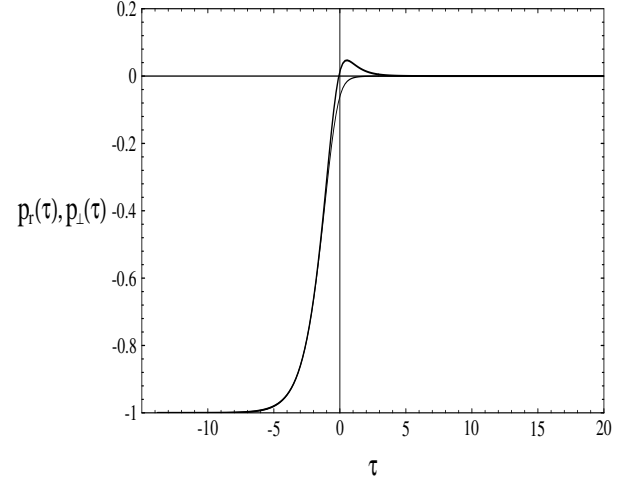


FIG. 4. Radial and tangential pressures $p_r < p_\perp$.

The equation of motion (16) for $f(R) = 0$ reduces to

$$\dot{\xi}^2 + 2\xi\ddot{\xi} - 3\xi^2 e^{-\xi^3} = 0 \quad (19)$$

It has the first integral

$$\dot{\xi}^2 = \frac{A - e^{-\xi^3}}{\xi} \quad (20)$$

and the second integral

$$\tau - \tau_0(R) = \int_{\xi_0}^{\xi} \sqrt{\frac{x}{A - e^{-x^3}}} dx \quad (21)$$

Here $\tau_0(R)$ is an arbitrary function (constant of integration parametrized by R) which is called the "bang-time function" [21]. For example, in the case of the Tolman-Bondi model for a dust, the evolution is described by $r(R, \tau) = (9GM(R)/2)^{1/3}(\tau - \tau_0(R))^{2/3}$, where $\tau_0(R)$ is an arbitrary function of R representing the big bang singularity surface for which $r(R, \tau) = 0$ [22].

The big bang starts from $\xi_0 = 0$. In our case this is the timelike regular surface $r = 0$ at the Fig.2. Choosing $\xi_0 = 0$ we fix the constant $A = 1$ and $\tau_0(R) = -R$. In coordinates (R, τ) the bang starts from the surface $R + c\tau = -\infty$ (see Fig.5).

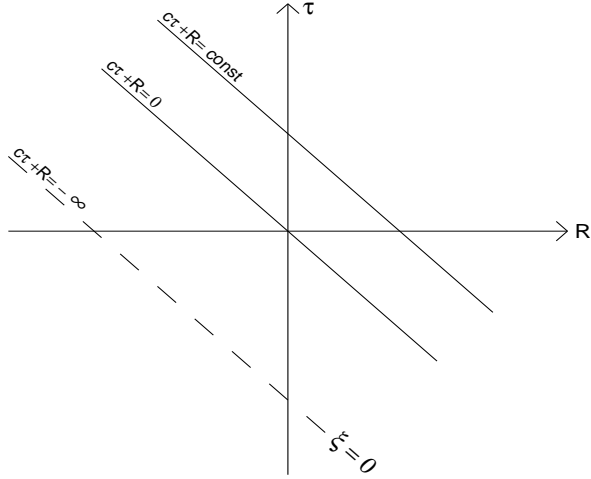


FIG. 5. The Lemaitre metric for a nonsingular white hole. Surfaces $r = \text{const}$ are plotted for the dimensionless radius ξ . The surface $\xi = 0$ is the big bang surface.

Different points of the bang surface $\xi = 0$ start at different moments of synchronous time τ . In the limit $\xi \rightarrow 0$ the law of the expansion is

$$\xi = e^{\tau - \tau_0(R)} = \dot{\xi} \quad (22)$$

This gives

$$e^\lambda = \frac{r^2}{r_0^2} \left(\frac{d\tau_0(R)}{dR} \right)^2 \quad (23)$$

and the metric takes the form

$$ds^2 = c^2 d\tau^2 - r_0^2 e^{\frac{2c\tau}{r_0}} (dq^2 + q^2 d\Omega^2), \quad (24)$$

where the variable $q = e^{\frac{R}{r_0}}$ is introduced to transform the metric into the FRW form. It describes, with the initial conditions $\xi_0(R + \tau \rightarrow -\infty) = 0$, $\dot{\xi}_0(R + \tau \rightarrow -\infty) = 0$, the nonsingular nonsimultaneous de Sitter bang.

In the case of a Schwarzschild WH, a singularity is spacelike (see Fig.1,3), so there exists the reference frame in which it is simultaneous. In the case of a Λ white hole, a regular surface $r = 0$ is timelike, and there does not exist any reference frame in which two events occurring on $r = 0$ would be simultaneous.

The first lesson of the Λ WH model is that the nonsingular de Sitter big bang must be nonsimultaneous.

The further evolution of the function ξ , velocity $\dot{\xi}$ and acceleration $\ddot{\xi}$ is shown in Fig. 6-8, obtained by numerical integration of the equation of motion (19) with the initial conditions $\xi_0 = 10^{-6}$, $\dot{\xi}_0 = 10^{-6}$.

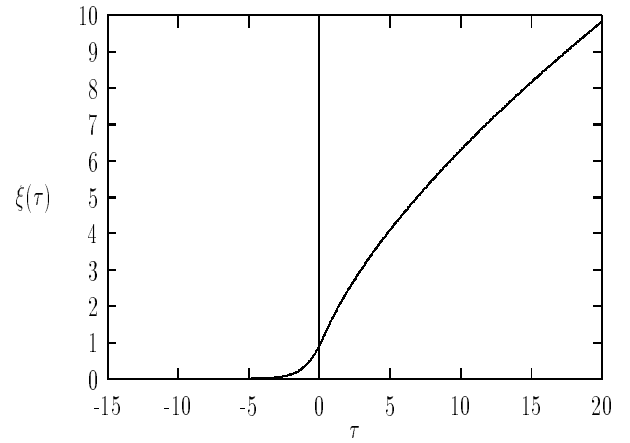


FIG. 6. The function $\xi(\tau - \tau_0)$ calculated from the equation of motion (19) with initial conditions $\xi_0 = \dot{\xi}_0 = 10^{-6}$.

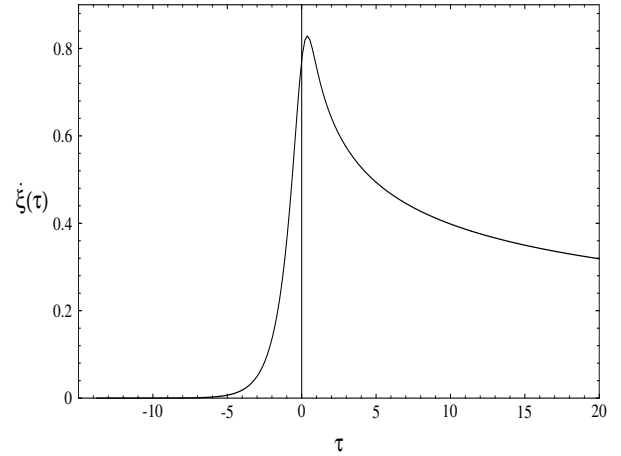


FIG. 7. The plot of the velocity $\dot{\xi}(\tau - \tau_0)$ for initial conditions $\xi_0 = \dot{\xi}_0 = 10^{-6}$.

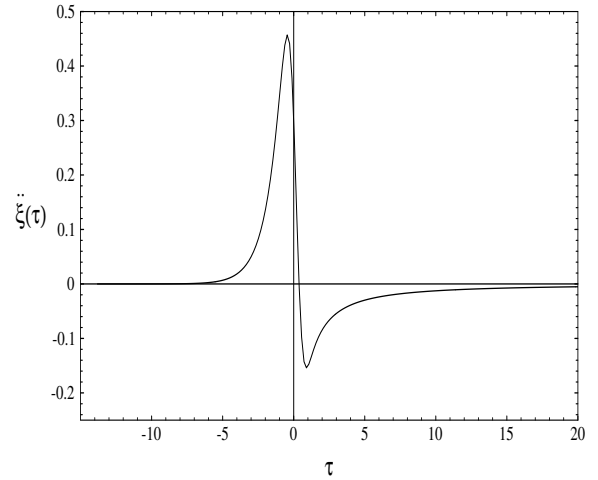


FIG. 8. The acceleration of the "scale factor" $\xi(\tau - \tau_0)$.

Numerical integration of the equation (19) shows an exponential growth of $\xi(\tau - \tau_0)$ at the beginning, when $p_\perp \simeq p_r \simeq -\rho$, followed by an anisotropic Kasner-type stage when the anisotropic pressure leads to an anisotropic expansion.

Qualitatively we can see this approximating the second integral (21) in the region $1 \ll \xi \ll (r_g/r_0)^{2/3}$ (far beyond a Λ WH horizon) by

$$\tau + R = \int_0^{\xi_0(R)} \sqrt{\frac{x}{1-e^{-x^3}}} dx + \int_{\xi_0(R)}^{\xi} \sqrt{x} dx$$

It gives

$$\xi = \left(\frac{9}{4}\right)^{\frac{1}{3}} (\tau + \tilde{\tau}_0(R))^{\frac{2}{3}}, \quad (25)$$

where $\tilde{\tau}_0(R) = R + \left(\frac{2}{3}\xi_0^{\frac{2}{3}}(R) - F(\xi_0(R))\right)$ and

$$F(\xi_0(R)) = \int_0^{\xi_0(R)} \sqrt{\frac{x}{1-e^{-x^3}}} dx$$

Then we get anisotropic Kasner-type metric

$$ds^2 = c^2 d\tau^2 - \left(\frac{9r_g}{4}\right)^{\frac{2}{3}} (\tau + \tilde{\tau}_0(R))^{-\frac{2}{3}} \left(\frac{d\tilde{\tau}_0(R)}{dR}\right)^2 dR^2 - \left(\frac{9r_g}{4}\right)^{\frac{2}{3}} (\tau + \tilde{\tau}_0(R))^{\frac{2}{3}} d\Omega^2 \quad (26)$$

with contraction in the radial direction and expansion in the tangential direction.

The second lesson of the Λ WH model is the existence of the anisotropic Kasner-type stage after inflation.

In our case the Kasner-type stage follows the stage of the nonsingular nonsimultaneous big bang from the regular surface $r = 0$. It looks that this kind of behaviour is generic for cosmological models near the origin [23] (for recent review see [24]). Our case differs from a singular case also in that the solution is not vacuum in the sense of zero source term in the Einstein equations, although it is vacuum in the sense that the variable cosmological term $\Lambda_{\mu\nu}$ corresponds to a spherically symmetric vacuum invariant under boosts in the radial direction [9,11].

Since the 3-curvature is zero for the case $f(R) = 0$, the Schwarzschild (ADM) mass \mathcal{M}_{ADM} coincides with the total proper (invariant) mass \mathcal{M}_{inv} which is the sum of the invariant masses of all particles with radial coordinates less than R given by [25]

$$\mathcal{M}_{inv}(R) = \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \int_0^R \rho \sqrt{-g} dR = 4\pi \int_0^R e^{\frac{\lambda}{2}} r^2 \rho dR$$

Indeed, for a fixed time section, r may be regarded as the function of R only [25]. Then $e^{\lambda/2} dR = dr$ and

$$\mathcal{M}_{inv} = 4\pi \int_0^r \rho x^2 dx = \mathcal{M}_{ADM} = \mathcal{M}(r)$$

At the beginning $\mathcal{M} = 0$ and $\dot{\mathcal{M}} = 0$, as it follows from the first integral (20) which gives

$$\dot{\xi}^2 = \frac{\mathcal{M}(\xi)}{\xi}$$

The behavior of a mass normalized to the Schwarzschild mass M is shown in the Fig.9.

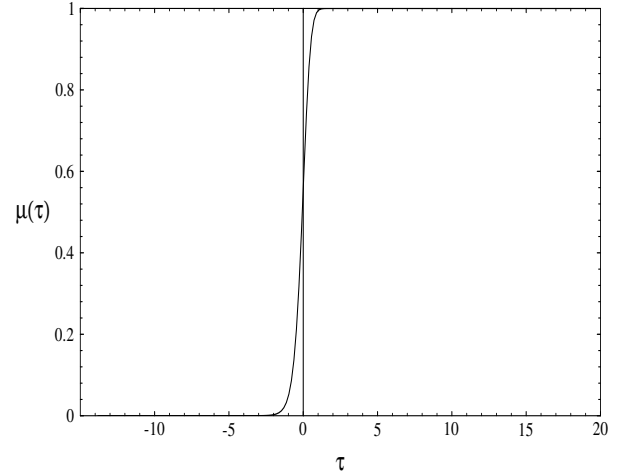


FIG. 9. Plot of the mass function $\mu = \mathcal{M}/M$.

During inflationary stage the mass increases as ξ^3 . At the next anisotropic stage the mass is growing abruptly towards the Schwarzschild mass M . Since the density is quickly falling at the same time starting from the initial value $\rho_0 = (8\pi G)^{-1}\Lambda$, the growth in a mass is connected with the fall of $\rho^\Lambda = (8\pi G)^{-1}\Lambda^t$, i.e., with the decay of the initial vacuum energy (the growth of a universe mass by many orders of magnitude in the course of decay of the de Sitter vacuum was first noticed in the Ref [26]).

The third lesson of the Λ WH model is the quick growth of the mass during the Kasner-type anisotropic stage.

Baby universes inside a Λ BH- In the case of direct de Sitter-Schwarzschild matching the global structure of spacetime (Fig.1) corresponds to arising of a closed or semiclosed world inside a BH [6]. In general case of a distributed density profile (the global structure of spacetime as shown in Fig.2) the physical situation near the timelike surface $r = 0$ is similar to that considered by Farhi and Guth. This region, which is the part of the regular core \mathcal{RC} , differs from that considered in Ref. [4,7] by an r -dependent density profile. Our density profile (18) is almost constant near $r \rightarrow 0$ and then quickly falls down to zero. In Fig.10 it is plotted for the case of a stellar mass black hole with $M = 3M_\odot$.

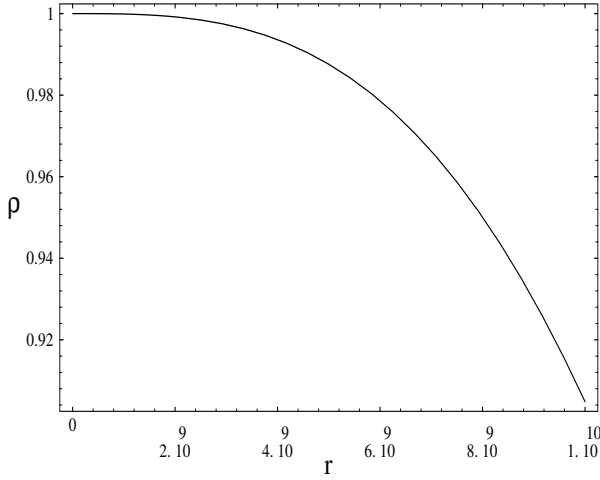


FIG. 10. Density profile (18) for the case of three solar masses black hole.

We may think of the region near $r = 0$ as of a small false vacuum bubble which can be a seed for a quantum birth of a new universe in accordance with the main idea of papers [4,7]. In this case the global structure of space-time is shown in Fig.11, which corresponds to arising of a closed or semiclosed world in one of the Λ WH structures in the future of a Λ BH in the original universe.

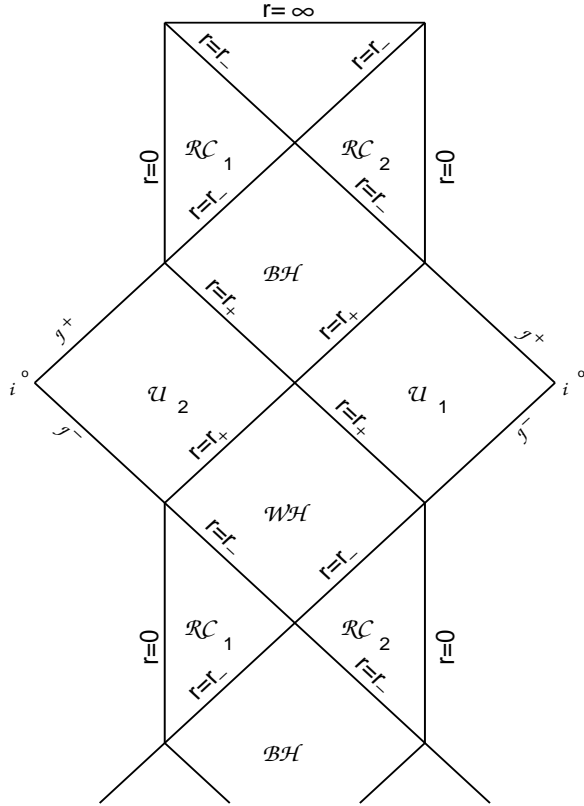


FIG. 11. The global structure of space-time for the case of a birth of a closed or semiclosed world inside a Λ black hole.

On the other hand, instability of a Λ white hole leads to possibilities other than considered by Farhi and Guth. Instabilities of Schwarzschild white holes are related to physical processes (particle creation) near a singularity (see, e.g., [15] and references therein). In the case of a Λ white hole its quantum instability is related to instability of the de Sitter vacuum near the surface $r = 0$. Instability of the de Sitter vacuum is well studied, both with respect to particle creation [27] and with respect to the quantum birth of a universe [26,28–35].

The possibility of a multiple birth of causally disconnected universes from the de Sitter background was noticed in 1975 in the Ref. [26]. In 1982 such a possibility has been investigated by Gott III who considered creation of a universe as a quantum barrier penetration leading to an open FRW cosmology [28]. The case of arising of open universes from de Sitter vacuum is illustrated by Fig.12 [28]. The events E and E' are creation of causally disconnected universes. The curved lines are world lines of comoving observers. At the spacelike surface AB the phase transition occurs from the inflationary to the radiation dominated stage [26,28].

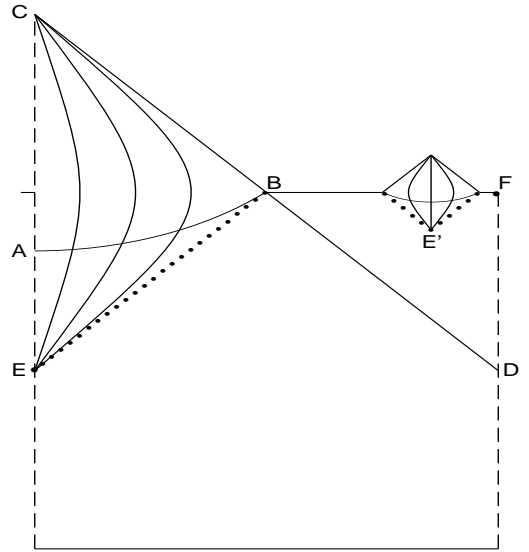


FIG. 12. Penrose-Carter diagram [28] corresponding to the case of a quantum birth of baby universes inside a Λ BH.

In the case of a Λ BH the region ECB in Fig.12 corresponds to the region \mathcal{RC}_1 in Fig.2, and the region BFD corresponds to a part of the region \mathcal{RC}_2 . The regions \mathcal{RC}_1 and \mathcal{RC}_2 in the de Sitter-Schwarzschild spacetime are entirely disjoint from each other for the same reason as the regions \mathcal{U}_1 and \mathcal{U}_2 (they can be connected by spacelike curves only). Birth of open (or flat) baby universes inside a Λ BH looks very similar to the picture shown in Fig.12. The essential difference is existence of an infinite number of the regions \mathcal{RC}_1 and \mathcal{RC}_2 inside a Λ BH.

In any case a nucleating spherical bubble can be described by a minisuperspace model with a single de

gree of freedom, the bubble radius [31,32], in our case $a = (r_0^2 r_g)^{1/3} \xi$.

The Friedmann equation in the conformal time ($cdt = ad\eta$) reads

$$\left(\frac{da}{d\eta}\right)^2 = \frac{8\pi G \rho a^4}{3c^2} - ka^2, \quad (27)$$

where $k = 0, \pm 1$. The standard procedure of quantization [31,32] results in the Wheeler-DeWitt equation in the minisuperspace for the wave function of universe [31] which reduces to the Schrödinger equation

$$\frac{\hbar^2}{2m_{Pl}} \frac{d^2\psi}{da^2} - [U(a) - E]\psi = 0 \quad (28)$$

with $E = 0$ and the potential (for the case of $k = 1$)

$$U(a) = \frac{m_{Pl} c^2}{2l_{Pl}^2} \left(a^2 - \frac{a^4}{r_0^2} \right) \quad (29)$$

With this equation we calculate the probability of a tunnelling which describes the quantum growth of an initial bubble on its way to the classically permitted region $a \geq r_0$, which corresponds to the case of a closed universe inside a black hole as in FG and FMM models [4,7,6].

This potential is plotted in Fig.13. It has two zeros, at $a = 0$ and $a = r_0$, and two extrema: the minimum at $a = 0$ and the maximum at $a = r_0\sqrt{2}$.

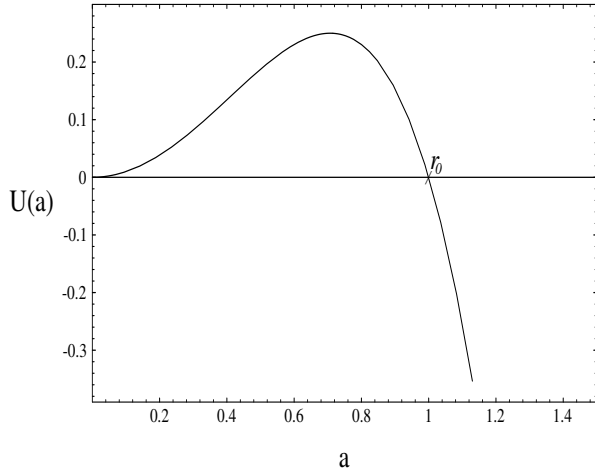


FIG. 13. Plot of the potential Eq.(29).

The WKB coefficient for penetration through the potential barrier reads [36]

$$D = \exp \left(-\frac{2}{\hbar} \left| \int_{a_1}^{a_2} \sqrt{2m_{Pl}[E - U(a)]} da \right| \right) \quad (30)$$

We get this probability in the frame of the Weeler-DeWitt equation, applying the general formula for the tunnel effect in quantum mechanics firstly calculated by Gamow [37]. In the Euclidean approach [38,31,32] the same result for the tunnelling probability is obtained [32] from

the WKB wave function with the Euclidean Action written in the imaginary time in the Euclidean domain (under a barrier where the kinetic energy term is negative). The sign in the exponent in Eq. (30) corresponds to the tunnelling wave function [31,32].

Calculating the integral in (30) we obtain

$$D = \exp \left[-\frac{2}{3} \left(\frac{r_0}{l_{Pl}} \right)^2 \right] \quad (31)$$

For the GUT scale $E_{GUT} \sim 10^{15} \text{ GeV}$ the probability of quantum birth of a universe is $D = \exp(-\frac{2}{3} 10^{16})$. This value of the penetration factor is in agreement with that calculated in the Ref. [7].

To estimate the probability of a quantum birth of an open or flat universe, we consider the instability of a Λ WH as evolved from a quantum fluctuation near $r = 0$ which contains some admixture of strings or quintessence [39] with the equation of state $p = -\rho/3$. In this case it is possible to find the nonzero probability of tunnelling for any value of k [40].

In the equation (27) the density evolves with a as

$$\rho = \rho_0 \left(\frac{a}{r_0} \right)^{-3(1+\alpha)}, \quad (32)$$

where α is a factor in the equation of state $p = \alpha\rho$. For the de Sitter vacuum $\alpha = -1$, and $\alpha = -1/3$ for strings or quintessence with $p = -\rho/3$. When both components are present in the initial fluctuation, the density can be written in the form [40]

$$\rho = \rho_0 \left(B_0 + B_2 \frac{r_0^2}{a^2} \right), \quad (33)$$

where B_0 and B_2 refers to the vacuum and strings (quintessence) contributions. The Friedmann equation (27) takes the form

$$\left(\frac{da}{d\eta}\right)^2 = (B_2 - k)a^2 + \frac{B_0 a^4}{r_0^2} \quad (34)$$

and transforms to the Schrödinger equation (28) with the potential

$$U(a) = \frac{m_{Pl} c^2}{2l_{Pl}^2} \left[(k - B_2)a^2 - \frac{B_0 a^4}{r_0^2} \right] \quad (35)$$

This potential has two zeros at $a_1 = 0$ and $a_2 = \sqrt{(k - B_2)/B_0} r_0$ and two extrema: the minimum at $a = 0$ and the maximum at $a = r_0 \sqrt{(k - B_2)/2B_0}$.

The WKB coefficient for penetration through the potential barrier (35) is given by

$$D = \exp \left(-\frac{2}{l_{Pl}} \left| \int_{a_1}^{a_2} \sqrt{(k - B_2)a^2 - \frac{B_0 a^4}{r_0^2}} da \right| \right) \quad (36)$$

We see that the presence of strings or a quintessence with the equation of state $p = -\rho/3$ in the initial fluctuation provides a possibility of quantum creation of flat ($k = 0$) and open ($k = -1$) universe with the probability

$$D = \exp \left(-\frac{2}{3} \left(\frac{r_0}{l_{pl}} \right)^2 \frac{\sqrt{(k - B_2)^3}}{B_0} \right) \quad (37)$$

For $r_0 \sim 10^{-25}$ cm, $D = \exp(-\frac{1}{3} \cdot 10^{16})$ for $k = 0$, $B_0 = 2$, $B_2 = -1$ which is very close to the value calculated above for the case of $k = 1$ and $B_2 = 0$, and to that obtained by Farhi, Guth and Guven [7].

The probability of a single tunnelling event is very small. However in the case of a Λ BH the probability of arising of a baby universe in a Λ BH is the probability of arising it in one of the Λ WH structures inside a Λ BH. Since there is an infinite number of Λ WH in one particular Λ BH, this probability is much greater than that for a single tunnelling event.

Conclusion - Let us emphasize that an obstacle related to the initial singularity does not arise in general case of a distributed profile, since both future and past singularities are replaced with the regular surfaces $r = 0$.

The results presented above concerning baby universes inside a Λ black hole are obtained for the case of an eternal black hole. In case a Λ BH is formed in the course of a gravitational collapse, the global structure of the space-time differs from that shown in Fig.2 by the presence of a matter. The analysis similar to that for the case of direct de Sitter-Schwarzschild matching [8], shows that estimates of probabilities of arising baby universes inside a Λ BH do not change for the case when it arises in a gravitational collapse. The probability of a quantum birth of baby universes inside a Λ black hole is not negligible due to existence of an infinite number of Λ WH structures inside each particular Λ black hole.

On the other hand, in the context of creation of a universe in the laboratory, the possibility of influence on the created universe is restricted by the presence of the Cauchy horizon in the de Sitter-Schwarzschild geometry.

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- [1] A.D.Sakharov, Sov. Phys. JETP **22**, 241 (1966)
[2] E.B.Gliner, Sov. Phys. JETP **22**, 378 (1966)
[3] M.R.Bernstein, Bull. Amer. Phys. Soc. **16**, 1016 (1984); W.Shen and S. Zhu, Phys. Lett. **A126**, 229 (1988)
[4] E.Farhi and A.Guth, Phys. Lett. **B183**, 149 (1987)
[5] E.Poisson and W.Israel, Class. Quant. Grav. **5**, L201 (1988)
[6] V.P.Frolov, M.A.Markov and V.F.Mukhanov, Phys. Lett. **B**, (1989)
[7] E.Farhi, A.Guth, and J.Guven, Nucl. Phys. **B339**, 417 (1990)
[8] V.P.Frolov, M.A.Markov and V.F.Mukhanov, Phys. Rev. **D41**, 3831 (1990)
[9] I.Dymnikova, Gen. Rel. Grav. **24**, 235 (1992)
[10] I.Dymnikova, Int. J. Mod. Phys. **D5**, 529 (1996)
[11] I.G.Dymnikova, Phys. Lett. **B 472**, 33 (2000)
[12] I.Dymnikova, in "Internal Structure of Black Holes and Spacetime Singularities", Ed. L. M. Burko and A. Ori, Inst. Phys. Publ., Bristol and Philadelphia, and The Israel Physical Society, p. 422 (1997)
[13] I.Dymnikova, Gravitation and Cosmology **5**, 15 (1999)
[14] S. W. Hawking and G. F. R. Ellis, "The Large Structure of Space-Time", Cambridge Univ. Press (1995)
[15] I.D.Novikov and V.P.Frolov, "Physics of Black Holes", Kluwer Academic Publishers (1989)
[16] I.Dymnikova, A.Magdziarz, B.Soltyszek, to be published
[17] L.D.Landau and E.M.Lifshitz, Classical Theory of Fields, Pergamon Press 1975
[18] R.Penrose, in "General Relativity: An Einstein Centenary Survey", Eds. S.W.Hawking and W.Israel, Cambridge Univ. Press, p. 581 (1979)
[19] R.C.Tolman, Proc. Nat. Acad. Sc. USA **20**, 169 (1934)
[20] M.G.Lemaitre, C.R.Acad. Sci. Paris **196**, 903 (1933)
[21] D.W.Olson, and J.Silk, Ap. J. **233**, 395 (1979)
[22] M.-N.Celerier, J.Schneider, Phys. Lett. **A249**, 37 (1998)
[23] V.A.Belinski, E.M.Lifshitz, I.M.Khalatnikov, Sov. Phys. Usp. **13**, 745 (1971)
[24] B.K.Berger, D.Garfinkle and V.Moncrief, in "International Structure of Black Holes and Spacelike Singularities", Eds. L.M.Burko and A.Ori, Inst. of Phys. Publishing, Bristol (1997)
[25] H.Bondi, MNRAS **107**, 410 (1947)
[26] E.B.Gliner, I.G.Dymnikova, Sov. Astr. Lett. **5**, (1975)
[27] N.D.Birrel and P.C.W.Davies, "Quantum Fields in Curved Space", Cambridge Univ. Press (1982); A.A.Grib, S.G.Mamayev, V.M.Mostepanenko, "Vacuum Quantum Effects in Strong Fields", Friedmann Lab. for Theor. Phys., St. Petersburg (1994)
[28] J.Richard Gott III, Nature **295**, 304 (1982).
[29] A.D.Linde, Phys. Lett. **108B**, 389 (1982)
[30] A.Albrecht and P.J.Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982)
[31] A.Vilenkin, Phys. Rev. **D30**, 509 (1984)
[32] A.Vilenkin, Phys. Rev. **D50**, 2581 (1994)
[33] A.D.Dolgov, Ya.B.Zel'dovich and M.V.Sazhin, "Cosmology of the Early Universe", Nauka, Moscow (1990).
[34] A.D.Linde, "Particle Physics and Inflationary Cosmology", Harwood, Switzerland (1990)
[35] Keith A.Olive, Phys. Rep. **190**, 307 (1990)
[36] L.D.Landau and E.M.Lifshitz, "Quantum Mechanics. Nonrelativistic Theory", Pergamon Press (1975)
[37] G. Gamow, Phys. Rev. **70**, 572 (1946)
[38] J. B. Hartle, S. W. Hawking, Phys. Rev. **D28**, 2960 (1983)
[39] R.R.Caldwell, R.Dave, P.J.Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998)
[40] M.L.Fil'chenkov, Phys. Lett. **B 354**, 208 (1995)